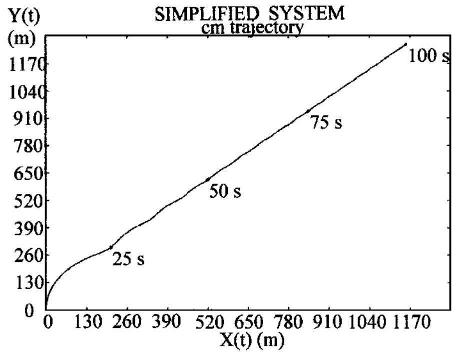

Fig. 3 Center-of-mass trajectory of a simplified system in planar motion.

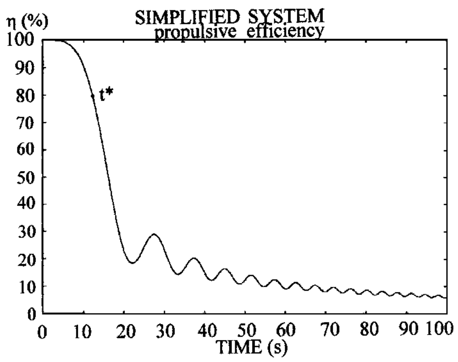

Fig. 4 Propulsive efficiency of a simplified system in planar motion.

interval of 12–14 m/s. From Eqs. (17) and (18) we get the convergence values $\dot{X}_\infty = \dot{Y}_\infty = 12.53$ m/s. The values t^* and $\dot{Y}^*$ for this example, obtained from Eqs. (23) and (25), are 12.53 s and 19.59 m/s, respectively. The actual value for $\dot{Y}^*$ is 19.55 m/s, indicating a relative error in the analytic model less than 0.5% (for this example). The ideal value of $\dot{Y}^*$, given by $(F/m)t^*$, is 25.07 m/s. Thus, the application of the force during t^* s ensures a propulsive efficiency, equal to the relation of $\dot{Y}^*$ and $(F/m)t^*$, greater than 77% in this case.

From Fig. 3 we can check that the approximate analytical solution agrees with the real solution for small t. The parabolic behavior of the c.m. trajectory described by Eq. (14) is clearly evident for small t. From Fig. 3 we also note that for large t the c.m. trajectory approaches the straight line described by Eqs. (19), (20), and (22). When $\varepsilon = 0$, the c.m. describes a straight trajectory in the Y direction, which in fact occurs in the first instants. The $\varepsilon \neq 0$ deviates this trajectory (the greater the value of ε then the faster the deviation) substantially.

From Fig. 4 we note that the propulsive efficiency η decreases monotonically with time until $\dot{Y}(t)$ reaches its first relative minimum, with $k = 3$ in Eq. (23).

Despite the numerical example being specific, Figs. 2–4 show the general behavior of the c.m. perturbations of a body subjected to the misalignment torque $F \cdot \varepsilon \cdot \hat{\boldsymbol{e}}_z$. In practice, we should avoid them by reducing ε, choosing parameters that increase t^*, thrusting during $t_{\text{on}} \ll t^*$, and by using a fast and precise attitude control.

Conclusions

This Note presented an analytical study of the effects of planar thrust misalignments on rigid body motion. Using some simplifications, we showed how the c.m. behaves in the absence of attitude control. We showed that 1) the c.m. motion degenerates quickly into an initial parabolic trajectory, 2) the thrust duration t_{on} should be much smaller than an upper bound t^*, and 3) the propulsive efficiency η falls quickly if t_{on} becomes comparable to t^*.

References

[1]Tandon, G. K., "Modeling Torques due to Orbit Maneuvers," *Astrophysics and Space Science Library*, Vol. 733, 1988, pp. 580–583.

[2]Schwende, M. A., and Strobl, H., "Bi-Propellant Propulsion Systems for Spacecraft Injection and Control," edited by C. Rowley and B. Battrick, *Attitude and Orbit Control Systems Proceedings*, European Space Agency, ESA SP-128, Paris, 1977, pp. 405–412.

[3]Longuski, J. M., Kia, T., and Breckenridge, W. G., "Annihilation of Angular Momentum Bias During Thrusting and Spinning-Up Maneuvers," *The Journal of the Astronautical Sciences*, Vol. 37, No. 4, 1989, pp. 433–450.

[4]Rodrigues, D. L. F., "Dynamic Analysis of Orbital Transfers," M.S. Thesis, Dept. of Space Mechanics and Control, National Inst. for Space Research, INPE-5352-TDI/461, São José dos Campos, Brazil, Oct. 1991.

[5]Gradshteyn, I. S., and Ryzhik, I. M., *Table of Integrals, Series and Products*, Academic International Press, New York, 1980, p. 395.

Generalized Canonical Systems Applications to Optimal Trajectory Analysis

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Introduction

THE coast-arc problem[1] that defines the optimal trajectory of a constant exhaust velocity space vehicle is described by a special class of systems of differential equations, termed generalized canonical systems.[2] Such systems have intrinsic properties concerning the Mathieu transformations defined by the general solution of the system of differential equations governed by the integrable kernel of the Hamiltonian function. By using these properties, integration of the system of differential equations describing the coast arc in a Newtonian central force field is performed. As will be shown, in contrast with other methods involving numerous integrations,[3,4] the generalized canonical approach requires the evaluation of only one integral, closely related to Kepler's classic equation. A complete closed-form solution will be obtained for elliptic, circular, parabolic and hyperbolic motions.

The coast-arc problem will be formulated as proposed by Powers and Tapley[5] through a two-dimensional formulation of the equations of motion. The three-dimensional case, considering a different set of state variables and orbital elements,[6,7] was previously discussed.

Coast-Arc Problem

For completeness, previous results about the coast-arc problem are presented.[6] Let us consider the motion of a space vehicle $\mathcal{M}$ in a Newtonian central force field during a coasting period. In a two-dimensional formulation,[5] the well-known equations of motion in polar coordinates are:

$$\frac{\mathrm{d}r}{\mathrm{d}t} = u, \qquad \frac{\mathrm{d}u}{\mathrm{d}t} = \frac{v^2}{r} - \frac{\mu}{r^2}, \qquad \frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{uv}{r}, \qquad \frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{v}{r} \tag{1}$$

where r is the radial distance from the center of attraction O; u and v are the radial and circumferential components of the velocity, respectively; θ is the polar angle, measured from any convenient reference line through the center of attraction; and μ is the gravitational parameter. The adjoint variables associated with the state variables $(r, u, v, \text{and } \theta)$ will be denoted by $(\pi_r, \pi_u, \pi_v, \text{and } \pi_\theta)$. The coast-arc problem is then described by the following Hamiltonian function:

$$H = u\pi_r + (v^2/r - \mu/r^2)\pi_u - (uv/r)\pi_v + (v/r)\pi_\theta \tag{2}$$

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The general solution of the state equations (1) is well known from the classic two-body problem,[8]

$$r = p/(1 + e\cos f), \qquad u = \sqrt{\mu/p}\, e\sin f$$
$$v = \sqrt{\mu/p}(1 + e\cos f), \qquad \theta = \omega + f \tag{3}$$

where p is the semilatus rectum, e is the eccentricity, ω is the pericenter argument, and f is the true anomaly. Equation (3) defines a point transformation, the inverse of which is

$$p = r^2v^2/\mu, \qquad e = \left\{1 + (2r^2v^2/\mu^2)\left[\tfrac{1}{2}(u^2 + v^2) - \mu/r\right]\right\}^{\frac{1}{2}}$$
$$f = \tan^{-1}[urv/(rv^2 - \mu)], \qquad \omega = \theta - \tan^{-1}[urv/(rv^2 - \mu)] \tag{4}$$

By following the properties of generalized canonical systems,[2] the general solution of the adjoint equations is given by the product between the Jacobian matrix of the inverse transformation (4) and the new adjoint vector $[\pi_p \;\; \pi_e \;\; \pi_f \;\; \pi_\omega]^T$. Therefore,

$$\pi_r = 2\frac{p}{r}\pi_p + \frac{\cos f + e}{r}\pi_e - \frac{\sin f}{re}(\pi_f - \pi_\omega)$$
$$\pi_u = \sqrt{\frac{p}{\mu}}\sin f\pi_e + \sqrt{\frac{p}{\mu}}\frac{\cos f}{e}(\pi_f - \pi_\omega)$$
$$\pi_v = 2\sqrt{\frac{p}{\mu}}r\pi_p + \sqrt{\frac{p}{\mu}}(2\cos f + e\cos^2 f + e)\frac{r}{p}\pi_e$$
$$- \sqrt{\frac{p}{\mu}}\frac{\sin f}{e}\left[1 + \frac{r}{p}\right](\pi_f - \pi_\omega)$$
$$\pi_\theta = \pi_\omega \tag{5}$$

The new Hamiltonian function resulting from the Mathieu transformation defined by Eqs. (3) and (5) is

$$\mathcal{H} = \left(\sqrt{\mu p}/r^2\right)\pi_f \tag{6}$$

The general solution of the new state equations is very simple,

$$p = p_0, \qquad e = e_0$$
$$\sqrt{\mu/p^3}(t - t_0) = J(f) - J(f_0), \qquad \omega = \omega_0 \tag{7}$$

where p_0, e_0, f_0, and ω_0 are arbitrary constants of integration, t_0 is the initial time, and $J(f)$ is

$$J(f) = -\frac{e\sin f}{(1 - e^2)(1 + e\cos f)}$$
$$+ \frac{2}{(1 - e^2)^{\frac{3}{2}}}\tan^{-1}\left(\sqrt{\frac{1 - e}{1 + e}}\tan\frac{f}{2}\right), \qquad e \neq 1 \tag{8}$$
$$J(f) = \tfrac{1}{2}\tan(f/2) + \tfrac{1}{6}\tan^3(f/2), \qquad e = 1 \tag{9}$$

For t_0 equal to the time of pericenter passage τ, the time equation in Eq. (7) reduces to Kepler's classic equation for elliptic orbits.[8] The general solution of the new adjoint equations is obtained as described before; thus,

$$\pi_p = \pi_{p0} + \frac{3}{2}\sqrt{\frac{\mu}{p^5}}(1 + e)^2(t - \tau)\pi_{f0}, \qquad \pi_f = \frac{(1 + e)^2}{p^2}r^2\pi_{f0}$$
$$\pi_e = \pi_{e0} - 2\sqrt{\frac{\mu}{p^3}}(1 + e)^2 I(f)\pi_{f0}, \qquad \pi_\omega = \pi_{\omega 0} \tag{10}$$

with $t_0 = \tau$, and

$$I(f) = \sqrt{\frac{p^3}{\mu(1 - e^2)^5}}\left[\sin E - e\left(M + \frac{1}{2}E + \frac{1}{4}\sin 2E\right)\right]$$
$$e < 1 \tag{11}$$

$$I(f) = \sqrt{\frac{p^3}{\mu(e^2 - 1)^5}}\left[\sinh F + e\left(M - \frac{1}{2}F - \frac{1}{4}\sinh 2F\right)\right]$$
$$e > 1 \tag{12}$$

$$I(f) = \left(1/4\sqrt{\mu}\right)\left[S - \tfrac{1}{3}D^3 - (1/5p)D^5\right], \qquad e = 1 \tag{13}$$

where $S = 2\sqrt{(\mu)}(t - t_0)$ and $D = \sqrt{(p)}\tan(f/2)$. M is the mean anomaly and E, F, and D are eccentric anomalies for elliptic, hyperbolic, and parabolic motions, respectively.

Therefore, from Eqs. (5) and (10), it follows that

$$\pi_r = 2\frac{p}{r}\pi_{p0} + \frac{\cos f + e}{r}\pi_{e0} + \frac{\sin f}{er}\pi_{\omega 0} + \sqrt{\frac{\mu}{p^3}}(1 + e)^2$$
$$\times \left[\frac{3}{r}(t - \tau) - \frac{2}{r}(\cos f + e)I(f) - \frac{r\sin f}{e\sqrt{\mu p}}\right]\pi_{f0} \tag{14}$$

$$\pi_u = \sqrt{\frac{p}{\mu}}\left\{\sin f\pi_{e0} - \frac{\cos f}{e}\pi_{\omega 0} + \sqrt{\frac{\mu}{p^3}}(1 + e)^2\right.$$
$$\left.\times \left[-2I(f)\sin f + \frac{r^2\cos f}{e\sqrt{\mu p}}\right]\pi_{f0}\right\} \tag{15}$$

$$\pi_v = \sqrt{\frac{p}{\mu}}\left\{2r\pi_{p0} + \frac{r}{p}(2\cos f + e\cos^2 f + e)\pi_{e0}\right.$$
$$+ \sqrt{\frac{\mu}{p^5}}(1 + e)^2 r\left[3(t - \tau) - 2I(f)(2\cos f + e\cos^2 f + e)\right.$$
$$\left.\left. - \frac{rp\sin f}{e\sqrt{\mu p}}\left(1 + \frac{r}{p}\right)\right]\pi_{f0} + \frac{\sin f}{e}\left(1 + \frac{r}{p}\right)\pi_{\omega 0}\right\} \tag{16}$$

$$\pi_\theta = \pi_{\omega 0} \tag{17}$$

The subscript 0 denoting the constants of integration was omitted for simplicity. The adjoint variables π_u and π_v are, respectively, the radial and circumferential components of the primer vector $\boldsymbol{p}_v$ introduced by Lawden[1] in the analysis of optimal space trajectories.

Note that Eqs. (14–16) have singularities for circular motion. To avoid these singularities, a new set of nonsingular orbital elements will be defined.

Elimination of Singularity

Let us consider the following set of orbital elements:

$$p' = p, \qquad h = e\cos\omega, \qquad k = e\sin\omega, \qquad \ell = \omega + f \tag{18}$$

The prime denotes the new variable. In terms of the new set of arbitrary parameters of integration (p', h, k, and ℓ), the general solution of state equations (1) is given by

$$r = p'/(1 + h\cos\ell + k\sin\ell), \qquad u = \sqrt{\mu/p'}(h\sin\ell - k\cos\ell)$$
$$v = \sqrt{\mu/p'}(1 + h\cos\ell + k\sin\ell), \qquad \theta = \ell \tag{19}$$

To obtain the adjoint variables (π_r, π_u, π_v, and π_θ) in terms of the new set of orbital elements and adjoint variables, we proceed as follows. First, we express the old adjoint variables (π_p, π_e, π_f, and π_ω) in terms of the new set of canonical variables ($p', h, k, \ell; \pi_{p'}, \pi_h, \pi_k$, and π_ℓ). After computing the Jacobian matrix of the point transformation (18), it follows that

$$\pi_p = \pi_{p'}, \qquad \pi_e = \pi_h\cos\omega + \pi_k\sin\omega$$
$$\pi_\omega = -\pi_h e\sin\omega + \pi_k e\cos\omega + \pi_\ell, \qquad \pi_f = \pi_\ell \tag{20}$$

Now, from Eqs. (5), (18), and (20)

$$\pi_r = 2(p'/r)\pi_{p'} + (1/r)(h+\cos\ell)\pi_h + (1/r)(k+\sin\ell)\pi_k$$

$$\pi_u = \sqrt{p'/\mu}\{\pi_h \sin\ell - \pi_k\cos\ell\}$$

$$\pi_v = \sqrt{p'/\mu}\{2r\pi_{p'} + (r/p')[(h\pi_h + k\pi_k) + (2 + h\cos\ell + k\sin\ell)(\pi_h\cos\ell + \pi_k\sin\ell)]\}$$

$$\pi_\theta = -k\pi_h + h\pi_k + \pi_\ell \qquad (21)$$

Equations (19) and (21) define a time-independent Mathieu transformation

$$(r, u, v, \theta; \pi_r, \pi_u, \pi_v, \pi_\theta) \xrightarrow{\text{Mathieu}} (p', h, k, \ell; \pi_{p'}, \pi_h, \pi_k, \pi_\ell)$$

The new Hamiltonian function resulting from this canonical transformation is

$$\mathcal{H} = \sqrt{\mu/p'^3}(1 + h\cos\ell + k\sin\ell)^2\pi_\ell \qquad (22)$$

The prime denoting the new variable p' will be omitted in the following paragraphs.

The general solution of the canonical system described by $\mathcal{H}$ is obtained using the same approach described in the preceeding section and is given by

$$p = p_0, \qquad h = h_0, \qquad k = k_0$$

$$\sqrt{\frac{\mu}{p^3}}(t-\tau) = \frac{2}{(1-h^2-k^2)^{\frac{3}{2}}}\tan^{-1}\left\{\sqrt{\frac{1-\sqrt{h^2+k^2}}{1+\sqrt{h^2+k^2}}}\tan\left(\frac{\ell-\ell_0}{2}\right)\right\} - \frac{(h\sin\ell - k\cos\ell)}{(1-h^2-k^2)(1+h\cos\ell+k\sin\ell)} \qquad (23)$$

$$\pi_p = \pi_{p_0} + \frac{3}{2}\sqrt{\frac{\mu}{p^5}}\left(1+\sqrt{h^2+k^2}\right)^2(t-\tau)\pi_{\ell_0}$$

$$\pi_h = \pi_{h_0} - 2\pi_{\ell_0}\frac{\left(1+\sqrt{h^2+k^2}\right)^2}{(1-h^2-k^2)^{\frac{5}{2}}}\left[-h\left(M+\frac{E}{2}\right) - \sqrt{1-h^2-k^2}\left(1-\sqrt{h^2+k^2}\right)\sin\ell_0 - \frac{1}{2\sqrt{1-h^2-k^2}}\left(\frac{r}{a}\right)\left\{(-2+2k^2+h^2)\sin\ell + hk\cos\ell + \left(\frac{r}{a}\right)\cos\ell(h\sin\ell - k\cos\ell)\right\}\right]$$

$$\pi_k = \pi_{k_0} - 2\pi_{\ell_0}\frac{\left(1+\sqrt{h^2+k^2}\right)^2}{(1-h^2-k^2)^{\frac{5}{2}}}\left[-k\left(M+\frac{E}{2}\right) + \sqrt{1-h^2-k^2}\left(1-\sqrt{h^2+k^2}\right)\cos\ell_0 + \frac{1}{2\sqrt{1-h^2-k^2}}\left(\frac{r}{a}\right)\left\{3hk\sin\ell + (2h^2-2-k^2)\cos\ell - \left(\frac{r}{a}\right)\sin\ell(h\sin\ell - k\cos\ell)\right\}\right]$$

$$\pi_\ell = \frac{\left(1+\sqrt{h^2+k^2}\right)^2}{(1+h\cos\ell+k\sin\ell)^2}\pi_{\ell_0} \qquad (24)$$

where ℓ_0 is the true longitude of the pericenter.

In view of Eqs. (18), Kepler's equation can be represented in the form

$$M = E - \left(1\big/\sqrt{1-h^2-k^2}\right)(r/a)(h\sin\ell - k\cos\ell) \qquad (25)$$

Therefore, from Eqs. (21), (24), and (25), it follows that

$$\pi_r = \frac{p}{r}\Bigg[2\pi_{p0} + \frac{1}{p}(h+\cos\ell)\pi_{h0} + \frac{1}{p}(k+\sin\ell)\pi_{k0} - \frac{2}{p}\pi_{\ell_0}\frac{\left(1+\sqrt{h^2+k^2}\right)^2}{(1-h^2-k^2)^{\frac{5}{2}}}\Bigg(-\frac{3}{2}M(1+h\cos\ell+k\sin\ell) + \sqrt{1-h^2-k^2}\left(1-\sqrt{h^2+k^2}\right)\sin(\ell-\ell_0) + \frac{1}{2\sqrt{1-h^2-k^2}}\left(\frac{r}{a}\right)\Bigg\{hk + (2-2h^2)(h\sin\ell - k\cos\ell) - k^2\sin 2\ell - hk\cos 2\ell - \left(\frac{r}{a}\right)\Bigg[h\sin\ell - k\cos\ell + \frac{1}{2}(h^2-k^2)\sin 2\ell - hk\cos 2\ell\Bigg]\Bigg\}\Bigg)\Bigg] \qquad (26)$$

$$\pi_u = \sqrt{\frac{p}{\mu}}\Bigg\{\sin\ell\pi_{h_0} - \cos\ell\pi_{k_0} - 2\pi_{\ell_0}\frac{\left(1+\sqrt{h^2+k^2}\right)^2}{(1-h^2-k^2)^{\frac{5}{2}}}\Bigg[\frac{3}{2}M(h\sin\ell + k\cos\ell) - \sqrt{1-h^2-k^2}\left(1-\sqrt{h^2+k^2}\right)\cos(\ell-\ell_0) + \frac{1}{2\sqrt{1-h^2-k^2}}\left(\frac{r}{a}\right)(2-2h^2-k^2 - hk\sin 2\ell + k^2\cos 2\ell)\Bigg]\Bigg\} \qquad (27)$$

$$\pi_v = \sqrt{\frac{p}{\mu}}\frac{r}{p}\Bigg(2p\pi_{p_0} + \left(\frac{3}{2}h + 2\cos\ell + \frac{k}{2}\sin 2\ell + \frac{h}{2}\cos 2\ell\right)\pi_{h_0} + \left(\frac{3}{2}k + 2\sin\ell + \frac{h}{2}\sin 2\ell - \frac{k}{2}\cos 2\ell\right)\pi_{k_0} + 2\pi_{\ell_0}\frac{\left(1+\sqrt{h^2+k^2}\right)^2}{(1-h^2-k^2)^{\frac{5}{2}}}\Bigg\{\frac{3}{2}M\Bigg[1 + \frac{1}{2}(h^2+k^2) + 2k\sin\ell + 2h\cos\ell + hk\sin 2\ell + \frac{1}{2}(h^2-k^2)\cos 2\ell\Bigg] - \sqrt{1-h^2-k^2}\left(1-\sqrt{h^2+k^2}\right)(2 + h\cos\ell + k\sin\ell)\sin(\ell-\ell_0) - \frac{1}{2\sqrt{1-h^2-k^2}}\left(\frac{r}{a}\right) \times \Bigg[2hk + (2-2h^2+k^2)h\sin\ell + \left(-2-\frac{1}{2}k^2+\frac{5}{2}h^2\right)k\cos\ell - 2k^2\sin 2\ell - 2hk\cos 2\ell - hk^2\sin 3\ell + \frac{k}{2}(k^2-h^2)\cos 3\ell - \left(\frac{r}{a}\right)[2h\sin\ell - 2k\cos\ell + (h^2-k^2)\sin 2\ell - 2hk\cos 2\ell]\Bigg]\Bigg\}\Bigg) \qquad (28)$$

$$\pi_\theta = -k\pi_{h_0} + h\pi_{k_0} - 2\pi_{\ell_0}\frac{\left(1+\sqrt{h^2+k^2}\right)^2}{(1-h^2-k^2)^{\frac{5}{2}}}$$
$$\times\left(\sqrt{(h^2+k^2)(1-h^2-k^2)}\left(1-\sqrt{h^2+k^2}\right)\right.$$
$$+\frac{1}{2\sqrt{1-h^2-k^2}}\left(\frac{r}{a}\right)\left\{2k(-1+2h^2+k^2)\sin\ell\right.$$
$$+2h(h^2-1)\cos\ell-\left(\frac{r}{a}\right)\left[\frac{1}{2}(h^2+k^2)+(1-h^2-k^2)^2\right.$$
$$\left.\left.\left.-hk\sin 2\ell+\frac{1}{2}(h^2-k^2)\cos 2\ell\right]\right\}\right) \tag{29}$$

where $r/a = (1-h^2-k^2)/(1+h\cos\ell + k\sin\ell)$. Except for the adjoint variables to the nonsingular orbital elements and the true longitude of the pericenter, the subscript denoting the constants of integration is omitted. For circular motion, Eqs. (26–29) simplify,

$$\pi_r = 2\pi_{p0} + (1/r)\left(\pi_{h_0} + 2\pi_{\ell_0}\sin\ell_0\right)\cos\ell$$
$$+(1/r)\left(\pi_{k_0} - 2\pi_{\ell_0}\cos\ell_0\right)\sin\ell + (3M/r)\pi_{\ell_0} \tag{30}$$

$$\pi_u = \sqrt{p/\mu}\left\{\left(\pi_{h_0}+2\pi_{\ell_0}\sin\ell_0\right)\sin\ell\right.$$
$$\left.-\left(\pi_{k_0}-2\pi_{\ell_0}\cos\ell_0\right)\cos\ell - 2\pi_{\ell_0}\right\} \tag{31}$$

$$\pi_v = \sqrt{p/\mu}\left\{2\left[p\pi_{p0}+(3M/2)\pi_{\ell_0}\right]+2\left(\pi_{h_0}+2\pi_{\ell_0}\sin\ell_0\right)\cos\ell\right.$$
$$\left.+2\left(\pi_{k_0}-2\pi_{\ell_0}\cos\ell_0\right)\sin\ell\right\} \tag{32}$$

$$\pi_\theta = \pi_{\ell_0} \tag{33}$$

Note that, for circular motion, ℓ_0 is the initial position of the space vehicle in orbit.

Conclusion

A closed-form solution of the coast-arc problem in a Newtonian central force field is obtained applying properties of generalized canonical systems for elliptic, circular, parabolic, and hyperbolic motions. This generalized canonical approach that involves a set of Mathieu transformations requires the evaluation of only one integral related to Kepler's classic equation.

References

[1]Lawden, D. F., "Fundamentals of Space Navigation," *Journal of British Interplanetary Society*, Vol. 13, March, 1954, pp. 87–101.

[2]Da Silva Fernandes, S., "Generalized Canonical Systems—I. General Properties," *Acta Astronautica*, Vol. 32, No. 5, 1994, pp. 331–338.

[3]Eckenwiler, M. W., "Closed-Form Lagrangian Multipliers for Coast Periods of Optimum Trajectories," *AIAA Journal*, Vol. 3, No. 6, 1965, pp. 1149–1151.

[4]Hempel, P. R., "Representation of the Lagrangian Multipliers for Coast Periods of Optimum Trajectories," *AIAA Journal*, Vol. 4, No. 4, 1966, pp. 729, 730.

[5]Powers, W. F., and Tapley, B. D., "Canonical Transformation Applications to Optimal Trajectory Analysis," *AIAA Journal*, Vol. 7, No. 3, 1969, pp. 394–399.

[6]Da Silva Fernandes, S., "Generalized Canonical Systems—III. Space Dynamics Applications: Solution of the Coast-Arc Problem," *Acta Astronautica*, Vol. 32, No. 5, 1994, pp. 347–354.

[7]Da Silva Fernandes, S., "A Note on the Solution of the Coast-Arc Problem," *Acta Astronautica* (to be published).

[8]Bate, R. R., Mueller, D. D., and White, J. E., *Fundamentals of Astrodynamics*, Dover, New York, 1971, pp. 51–226.